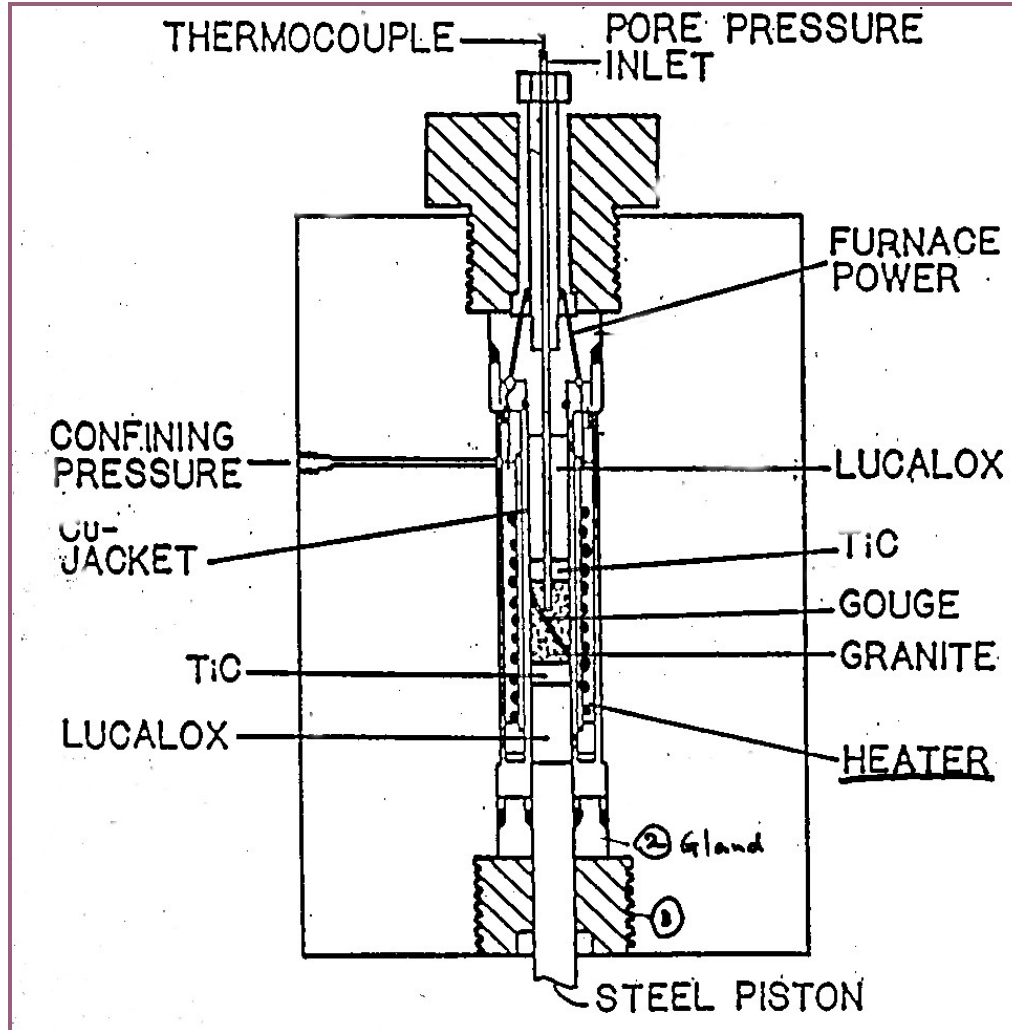


Pressure vessel

Ref :T. **Shimamoto, Seminar design (2013)**, Tullis and Tullis, Experimental deformation techniques (1986), Paterson and Wong Experimental rock déformation: the brittle field (2005), ...



(MIT group; D. E. Moore et al., 1989, JSG, 11: 329-341)

A pressure vessel for gas rig

(1) Pressure vessel

(3) Nut

*Most vessel failure started from the first few threads of nuts.
(L. M. Logan)*

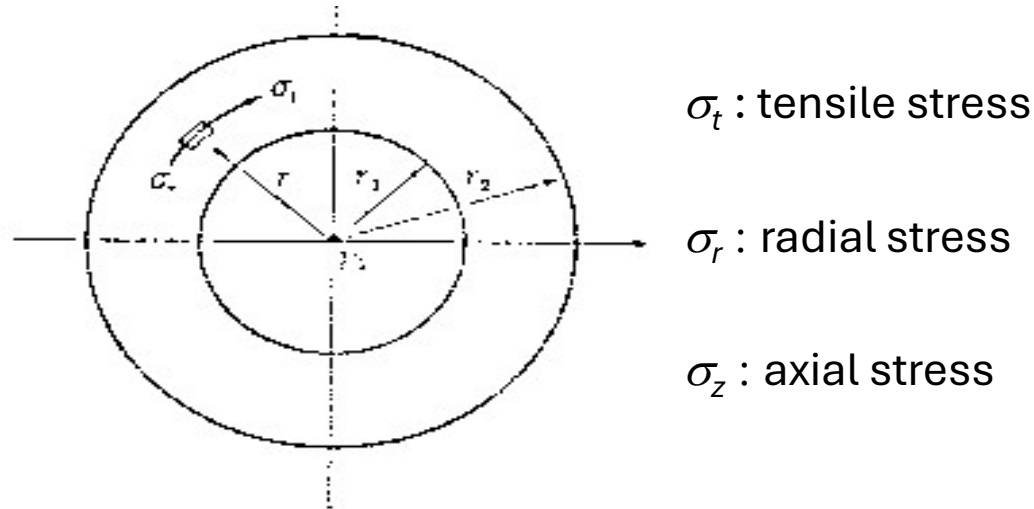
(4) Piston & spacers
--- *split piston!!*

(5) Furnace & T/C

(6) Specimen & jacket

Stresses for an internally pressurized cylinder

Stresses for an internally
pressurized cylinder



σ_t : tensile stress

σ_r : radial stress

σ_z : axial stress

The maximum tensile
stress at the inner wall

$$\sigma_t^{max} = \left[\frac{(\kappa^2 + 1)}{(\kappa^2 - 1)} \right] P_i$$

with $\kappa = r_o/r_i$

r_o, r_i : external & internal diameters

$$\sigma_r = -\frac{P_i}{\kappa^2 - 1} \left\{ \left(\frac{r_2}{r} \right)^2 - 1 \right\} \text{ kg/cm}^2$$

$$\sigma_t = \frac{P_i}{\kappa^2 - 1} \left\{ \left(\frac{r_2}{r} \right)^2 + 1 \right\} \text{ kg/cm}^2$$

$$\sigma_z = -\frac{P_i}{\kappa^2 - 1} \text{ kg/cm}^2$$

Example:

for $r_o/r_i = 3$, $\sigma_t^{max} = 1.25 P_i$

(25 % increase in σ_t^{max} due to P_i)

Stresses for an internally pressurized cylinder

The maximum tensile
Stress at the inner wall
(most dangerous!)

$$\sigma_t^{max} = \left[\frac{(\kappa^2 + 1)}{(\kappa^2 - 1)} \right] P_i$$

with $\kappa = r_o/r_i$

$$r_o/r_i = 2, \quad \sigma_t^{max} = 1.67 P_i$$

$$r_o/r_i = 3, \quad \sigma_t^{max} = 1.25 P_i$$

$$r_o/r_i = 4, \quad \sigma_t^{max} = 1.13 P_i$$

$$r_o/r_i = 5, \quad \sigma_t^{max} = 1.08 P_i$$

$$r_o/r_i = 10, \quad \sigma_t^{max} = 1.02 P_i$$

(1) Select specimen size and
determine r_i .

(2) Select material for vessel

Example **AISI E4347** ---

Tensile strength = ca. 1 GPa

(3) Select outer diameter r_o and
determine pressure limit
with safety factor of 4.

Example: for $r_o/r_i = 3$

$$P_i^{max} = (1 \text{ GPa}/4)/1.25 \\ = 200 \text{ MPa}$$

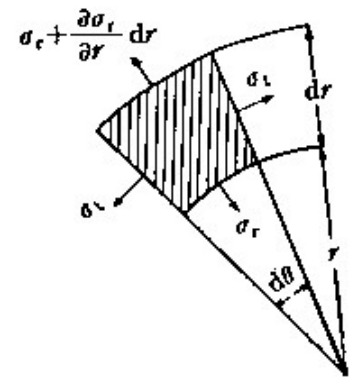
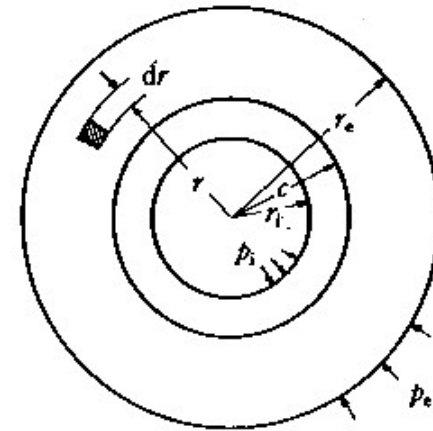
*This simple method gives about two-
times safety for internal yielding.*

Internally and Externally Pressurized Cylinder (THICK WALL)

(Tensile stresses are taken as positive.)

$$\left. \begin{aligned} \sigma_t &= \frac{-p_e K^2 + p_i}{K^2 - 1} - \frac{(p_e - p_i) K^2}{K^2 - 1} \left(\frac{r_i}{r} \right)^2 \\ \sigma_r &= \frac{-p_e K^2 + p_i}{K^2 - 1} + \frac{(p_e - p_i) K^2}{K^2 - 1} \left(\frac{r_i}{r} \right)^2 \\ \sigma_z &= \frac{1}{K^2 - 1} p_i \quad \sigma_z = 0 \end{aligned} \right\}$$

(with confined ends) (with free ends)



Internally and Externally Pressurized Cylinder

(Tensile stresses are taken as positive.)

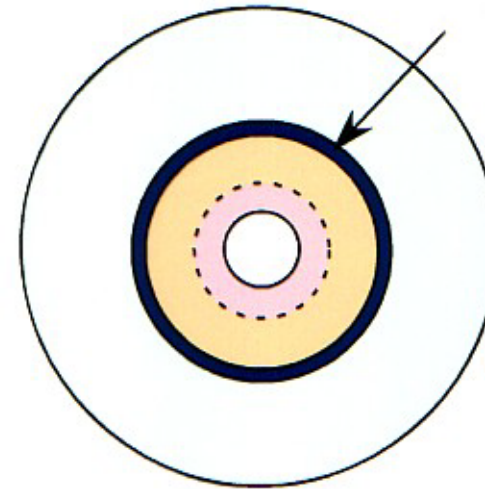
$$\left. \begin{aligned} \sigma_t &= -\frac{p_e K^2 + p_i}{K^2 - 1} - \frac{(p_e - p_i) K^2}{K^2 - 1} \left(\frac{r_i}{r} \right)^2 \\ \sigma_r &= -\frac{p_e K^2 + p_i}{K^2 - 1} + \frac{(p_e - p_i) K^2}{K^2 - 1} \left(\frac{r_i}{r} \right)^2 \\ \sigma_z &= \frac{1}{K^2 - 1} p_i \end{aligned} \right\}$$

(with confined ends) (with free ends) $\sigma_z = 0$

$$\sigma_t^{max} = \left[\frac{(\kappa^2 + 1)}{(\kappa^2 - 1)} \right] p_i - \left[\frac{2\kappa^2}{(\kappa^2 - 1)} \right] p_e$$

For $\kappa = 3$, $\sigma_t^{max} = 1.25 p_i - 2.25 p_e$

Externally pressurized
double-walled
pressure vessel



$P_{oil} = \text{ca. } 500 \text{ MPa}$
can make it possible
to sustain $p_i > 1 \text{ GPa}$.

Plastic Deformation of Pressure Vessel

Tresca criterion:

$$\sigma_1 - \sigma_3 = 2\tau_y = \sigma_y$$

von Mines Criterion:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_y^2$$

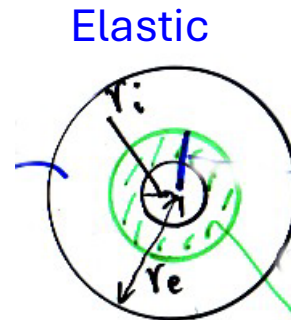
Initial yielding (Tresca):

$$p_y = \frac{\sigma_y}{2} \left(1 - \frac{1}{K^2} \right)$$

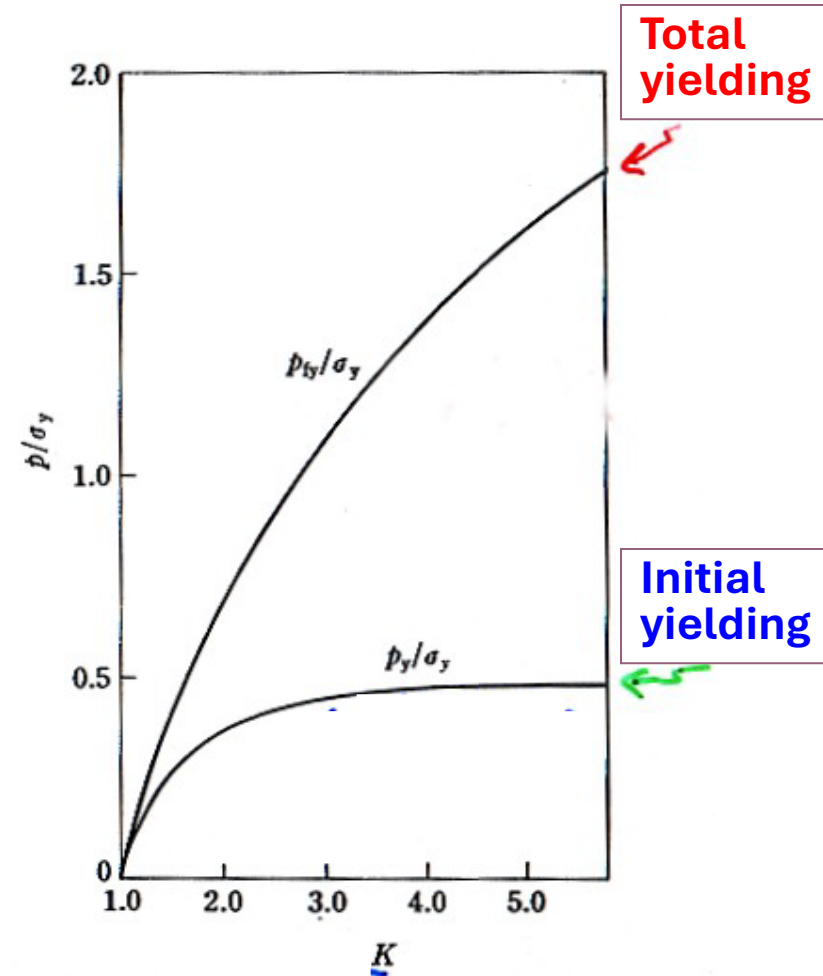
Total yielding (Tresca):

$$p_{ty} = \sigma_y \ln K$$

Solid-pressure medium rig is used close to this limit!



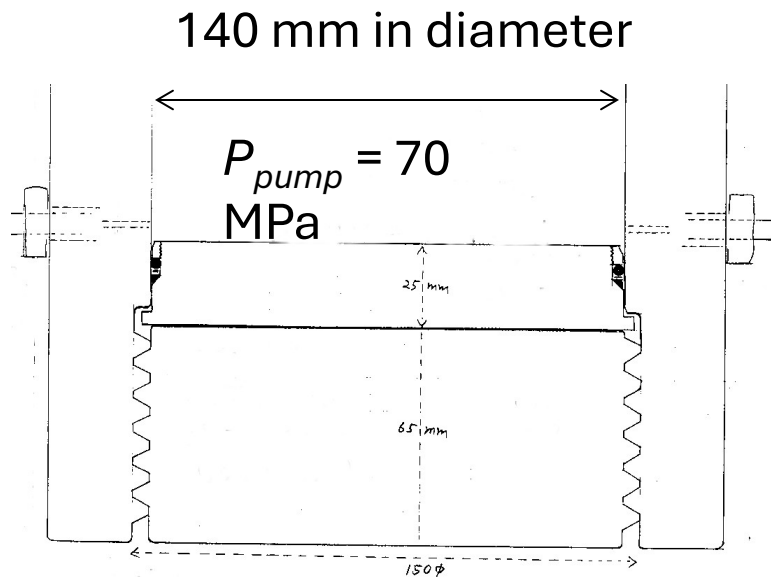
Plastically deformed zone



*Heard's used it
But NOT EU*

Designing Nuts

‘I saw more than 10 failed pressure vessels. In most cases, failure started from the first few threads of Nuts. It is best to design nuts so only one thread sustains forces acting on it. (*J. M. Logan*)’



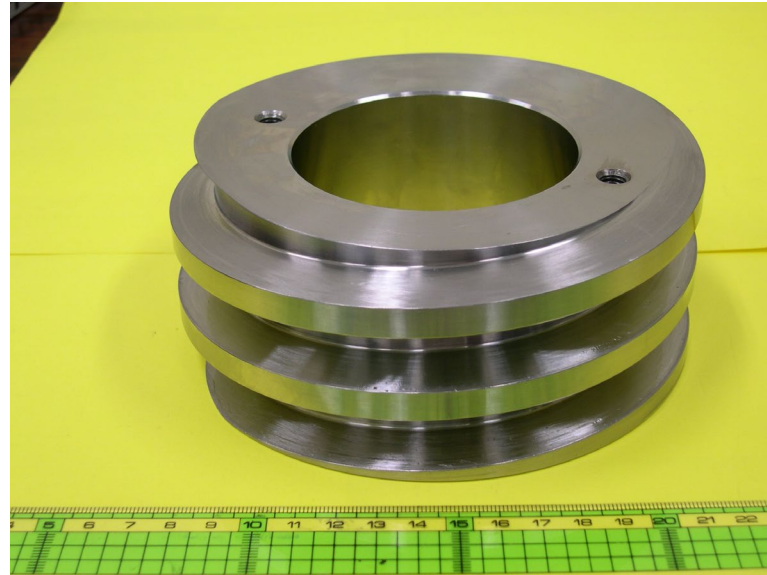
Area = 154 cm², $F = 108$ tons

$$\frac{108 \text{ tons}}{44 W} = 10/4 \text{ Kb}$$

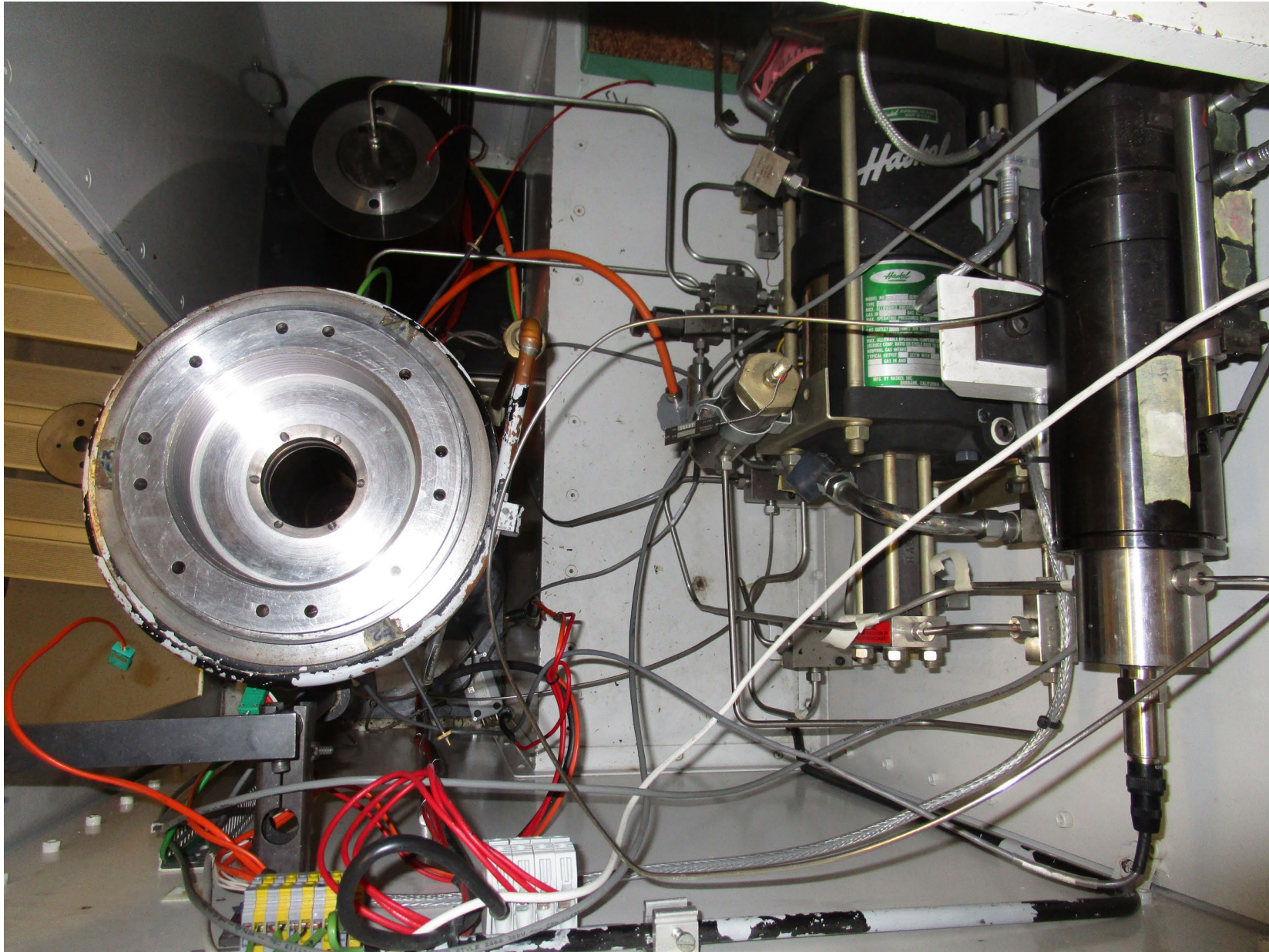
- (1) Determine the wall thickness of the nut. **In many cases, this is taken too small!**
- (2) Calculate force acting on nut.
- (3) Determine the base width of nut with 4 times safety to support the force with one thread.
Trapezoidal threads are better!

Circumference = $14 \times 3.14 = 44\text{cm}$
 Take shear strength as 1 GPa = 10 Kb.
 Then with 4 times safety the width of thread W has as to be 0.98 cm = 9.8 mm.

Upper nut can sustain
about
100 ton force.



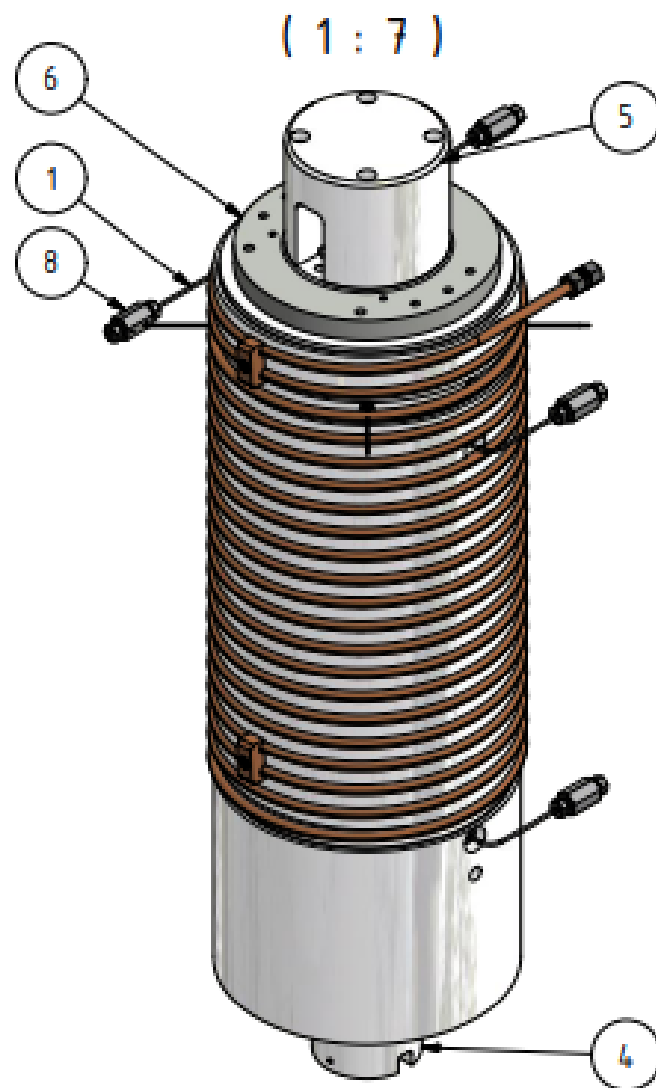
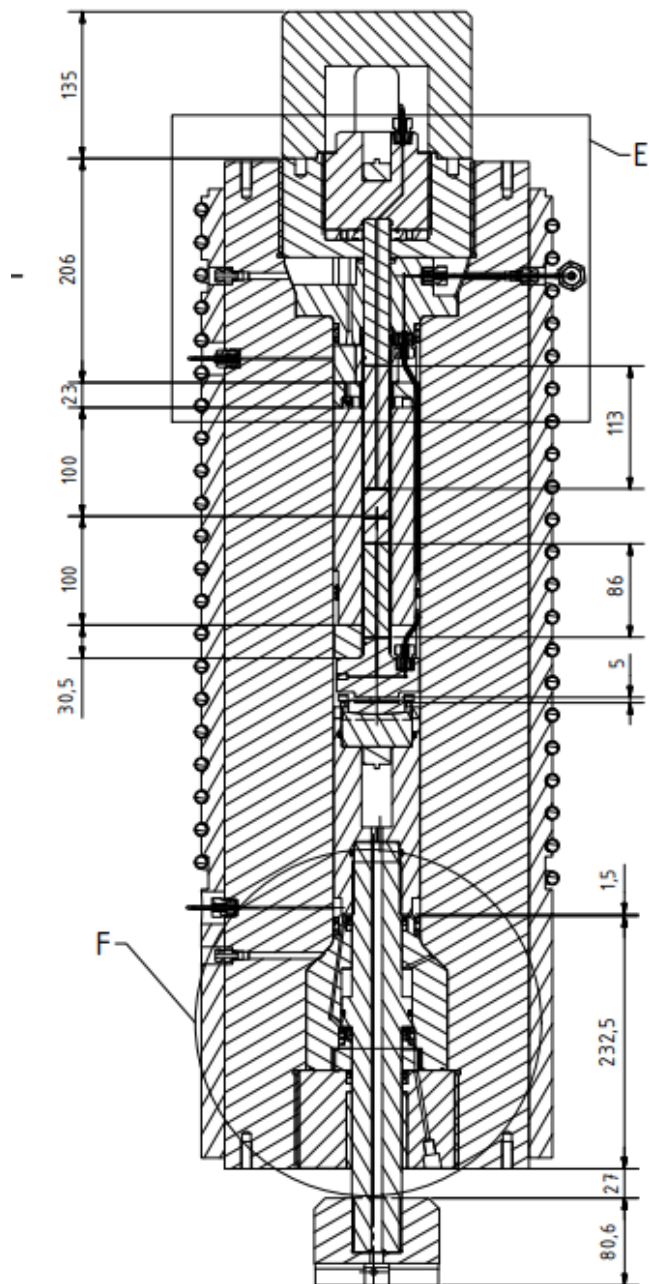
Be careful for violent fracturing experiments!
The assembly is ideal for gas rig.



Paterson machine Montpellier



Paterson machine Montpellier



Target EPFL